Quantifying the errors of the coupling schemes at the ocean-atmosphere interface in Earth system models, using a mathematically consistent Schwarz iterative method

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- **Coupling ocean and atmosphere : theory and reality in coupled ocean-atmosphere model**
  - time scheme(s)
- **Quantifying time scheme error with a Schwarz iterative method**
- **Impact on short experiments**
- **And now ?**

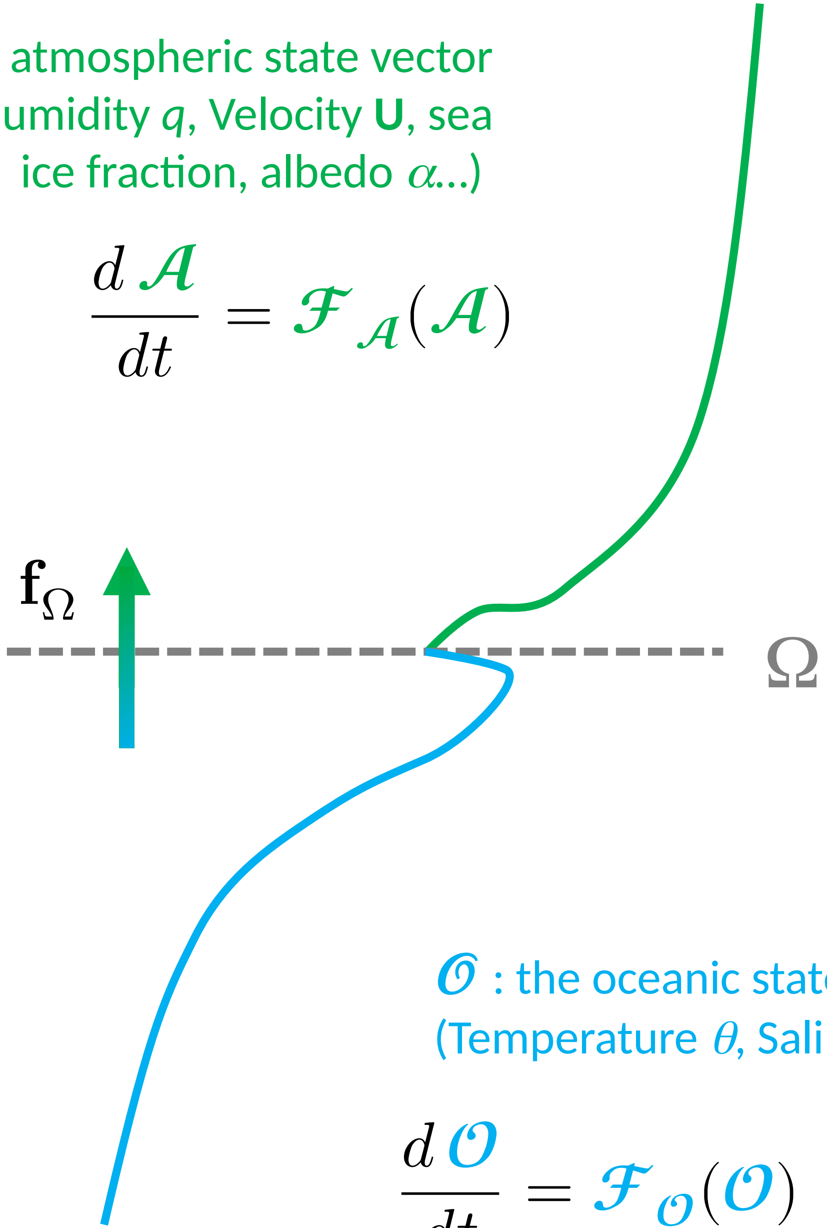
Coupling ocean and atmosphere : theory and reality in coupled ocean-atmosphere models

Air-sea coupling in theory

$\mathscr{A}$: the atmospheric state vector
(Temperature θ, Humidity q, Velocity **U**, sea ice fraction, albedo α...)

$$\frac{d\,\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A})$$

Fluxes accross the boundary Ω
Heat
Water
Momentum
...



Coupling = continuity at interface Ω

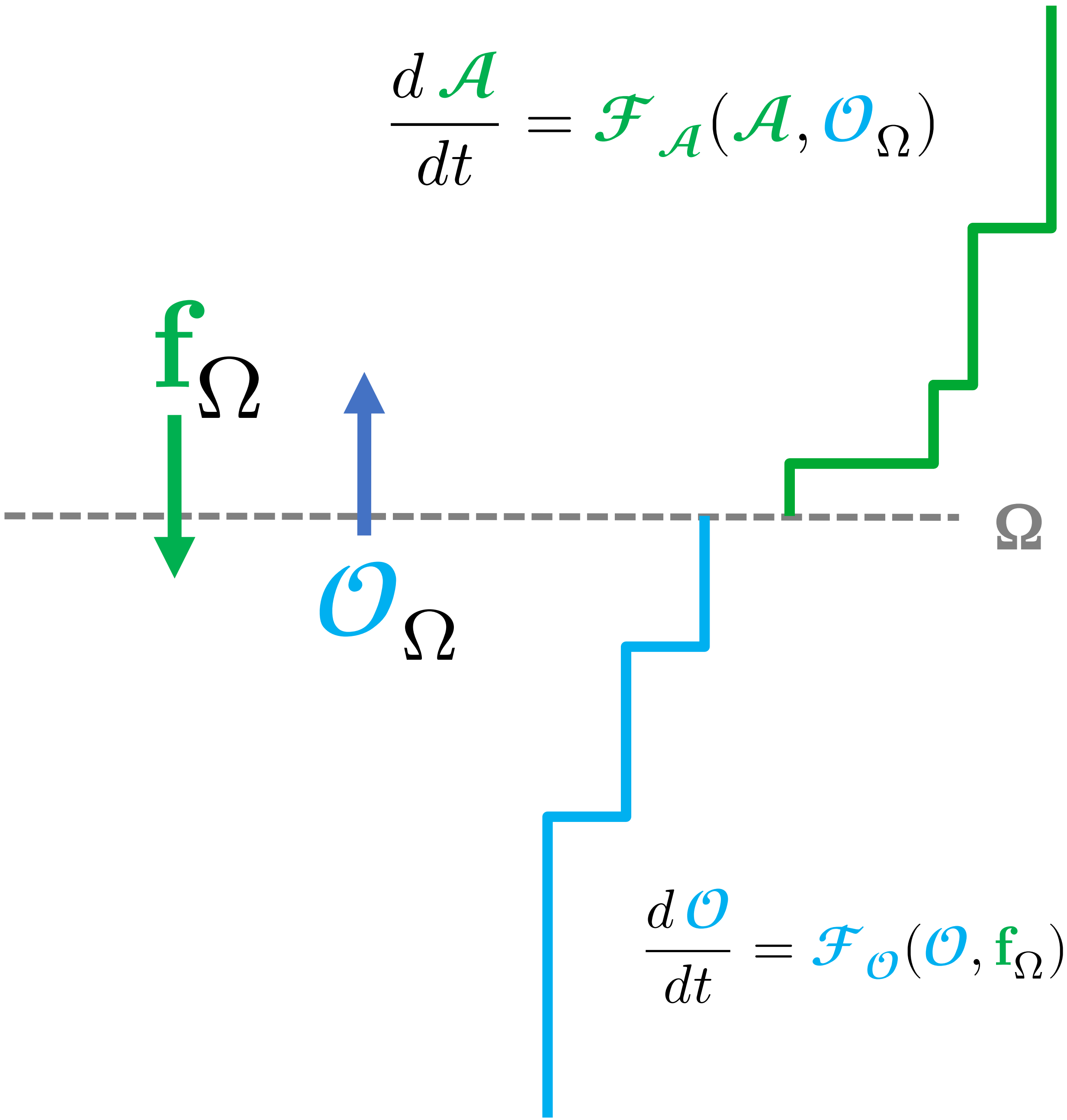
$$\begin{cases} \mathcal{A}_\Omega = \mathcal{O}_\Omega \\ \left.\dfrac{\partial\,\mathcal{A}}{\partial z}\right|_\Omega = \left.\dfrac{\partial\,\mathcal{O}}{\partial z}\right|_\Omega = \mathbf{f}_\Omega \end{cases}$$

$\mathscr{O}$: the oceanic state vector
(Temperature θ, Salinity S, Velocity **U**, ...)

$$\frac{d\,\mathcal{O}}{dt} = \mathcal{F}_{\mathcal{O}}(\mathcal{O})$$

Air-sea coupling in ocean-atmosphere models

$$\frac{d\,\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A}, \mathcal{O}_{\Omega})$$



$$\frac{d\,\mathcal{O}}{dt} = \mathcal{F}_{\mathcal{O}}(\mathcal{O}, \mathrm{f}_{\Omega})$$

$\mathcal{O}_{\Omega}$ is estimated with
$\mathcal{O}(k{=}1) = \mathcal{O}(z{=}0.5 \text{ to } 5 \text{ m})$

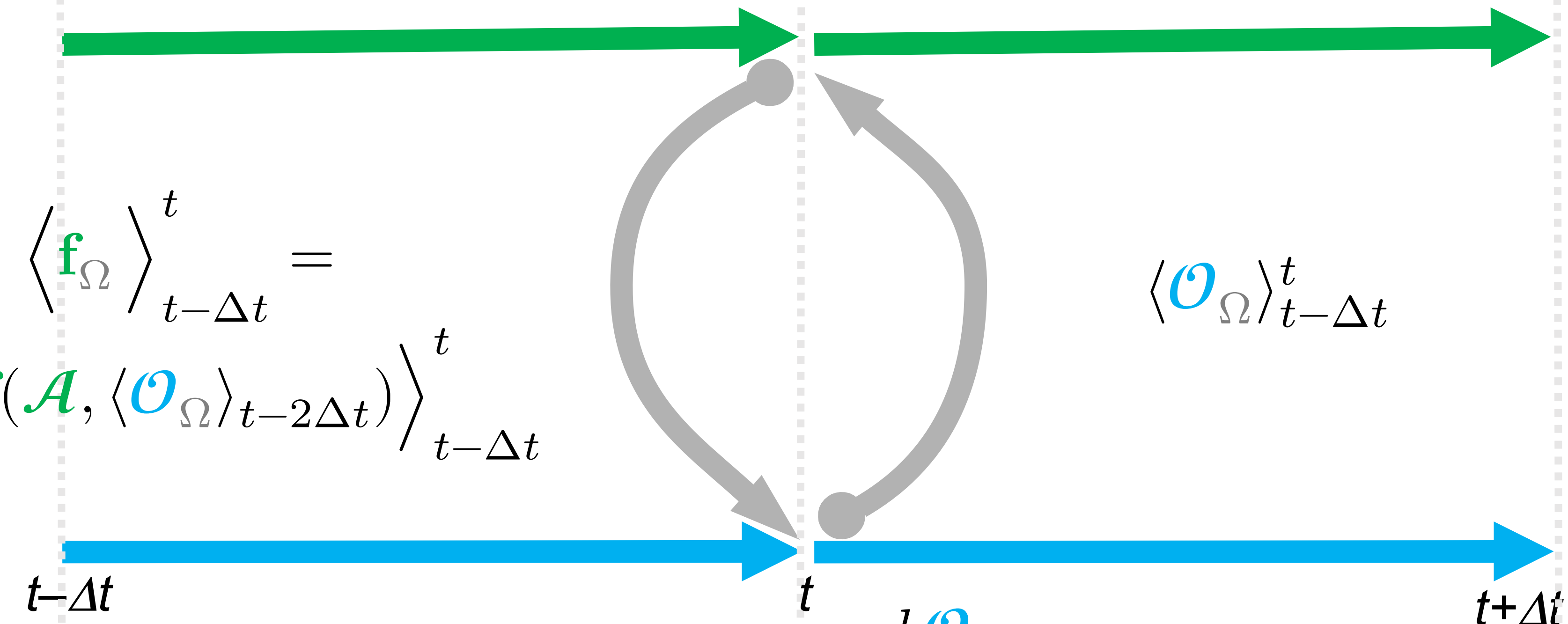
Time stencil of the exchanges between ocean and atmosphere : Parallel

[t, $t+\Delta t$] is the coupling time step. $\langle \bullet \rangle$ is the time average between to times

$$\frac{d\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A}, \langle \mathcal{O}_\Omega \rangle_{t\text{-}\Delta t}^{t})$$

$$\left\langle \mathbf{f}_\Omega \right\rangle_{t-\Delta t}^{t} = \left\langle \mathbf{f}(\mathcal{A}, \langle \mathcal{O}_\Omega \rangle_{t-2\Delta t}) \right\rangle_{t-\Delta t}^{t}$$

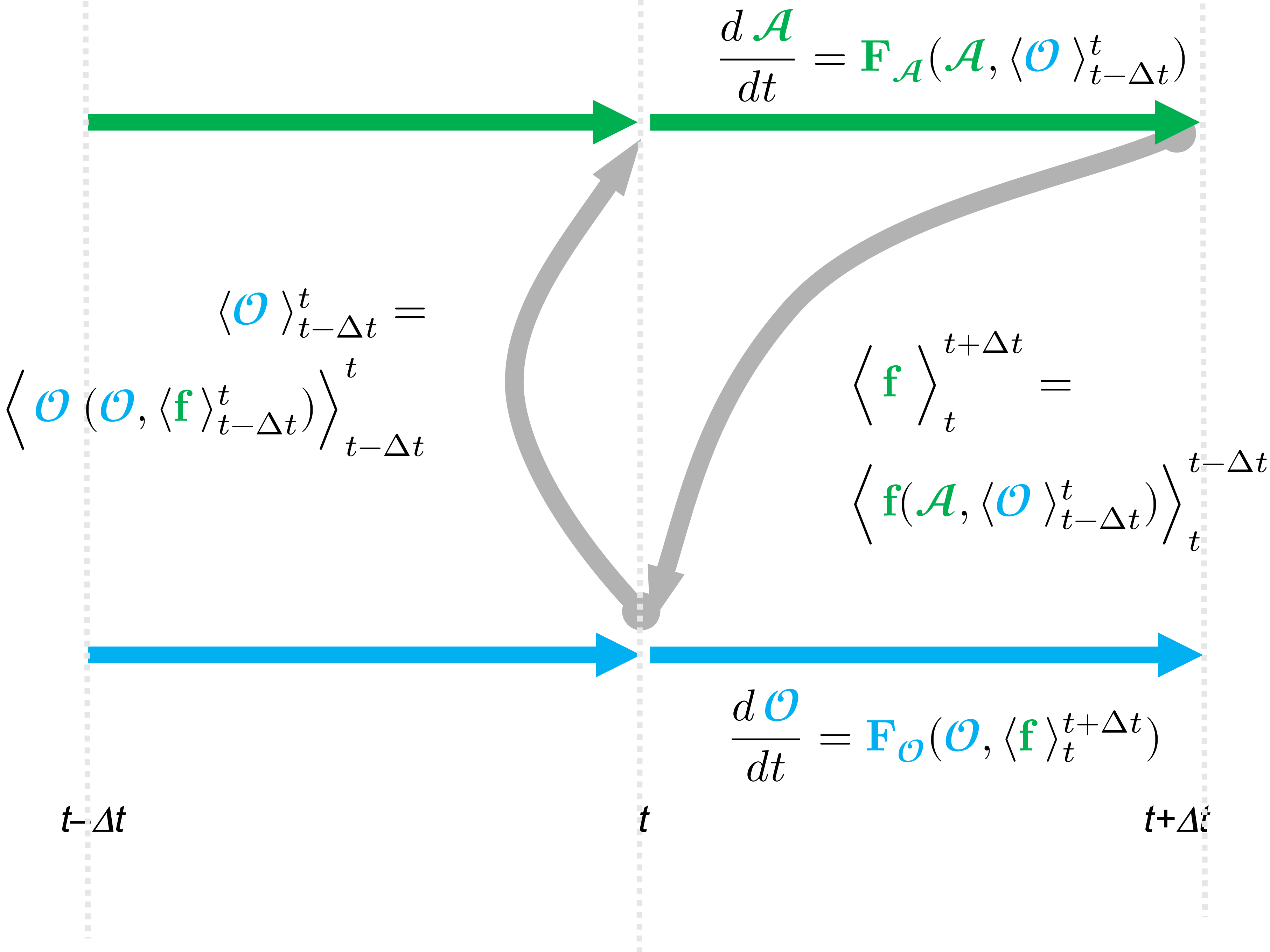
$$\langle \mathcal{O}_\Omega \rangle_{t-\Delta t}^{t}$$



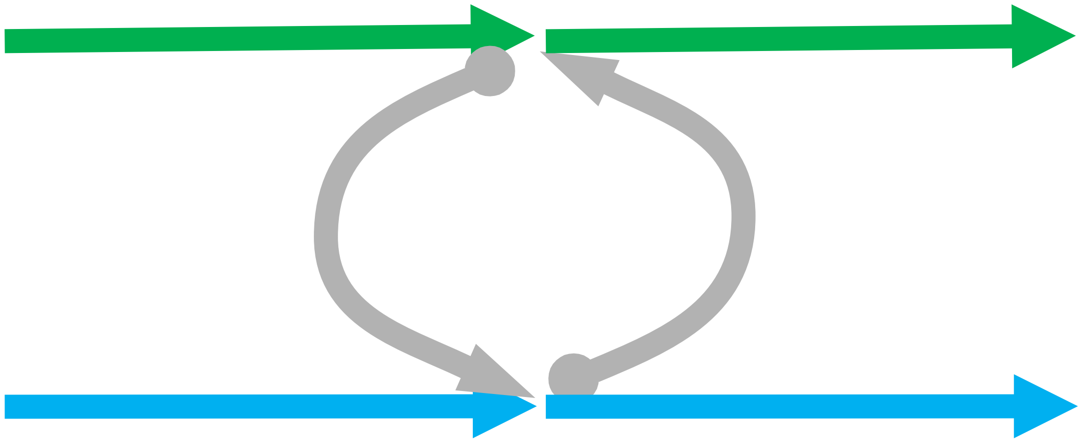
$$\frac{d\mathcal{O}}{dt} = \mathcal{F}_{\mathcal{O}}(\mathcal{O}, \langle \mathbf{f}_\Omega \rangle_{t\text{-}\Delta t}^{t})$$

Redrawn from Fig. 5.4, Lemarié et al. (2008)

Sequential atmosphere first

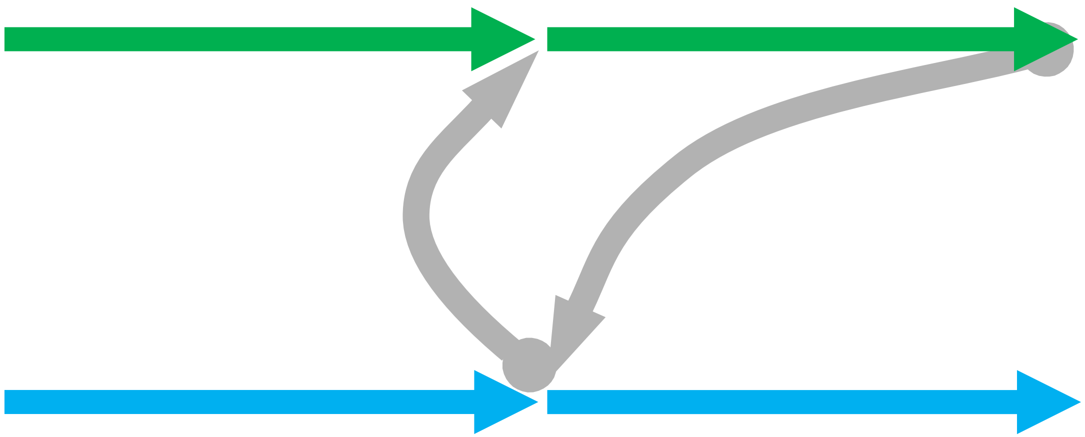


Parallel



IPSL-CM6, CNRM-CM6-1, EC-Earth3, MPI-ESM, HadGEM3-GC31, ?

Sequential Atmosphere First



ECMWF, RPN, ?

Mathematical inconsistency of state-of-the-art ocean atmosphere models

$$\frac{d\,\mathcal{O}}{dt}\bigg|_{t}^{t+\Delta t} = \mathcal{F}_{\mathcal{O}}\left(\mathcal{O}, \left\langle \mathbf{f}(\mathcal{A}, \langle \mathcal{O}_{\Omega} \rangle_{t-2\Delta t}^{t-\Delta t}) \right\rangle_{t-\Delta t}^{t}\right)$$

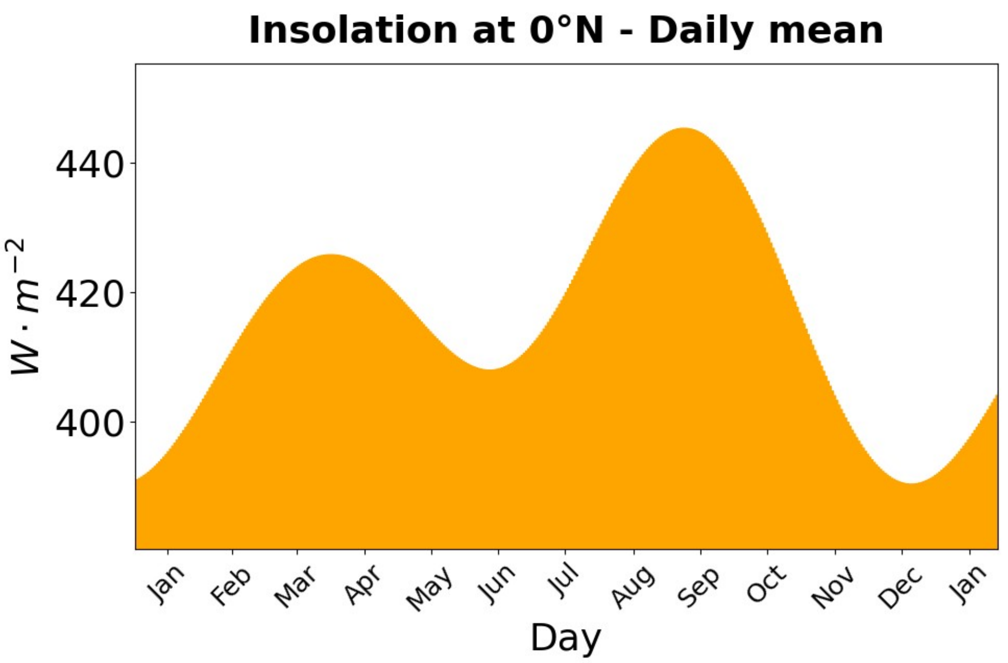
Time schemes may differ between model, but no model uses a synchronous time stepping, no model is mathematicaly consistent.

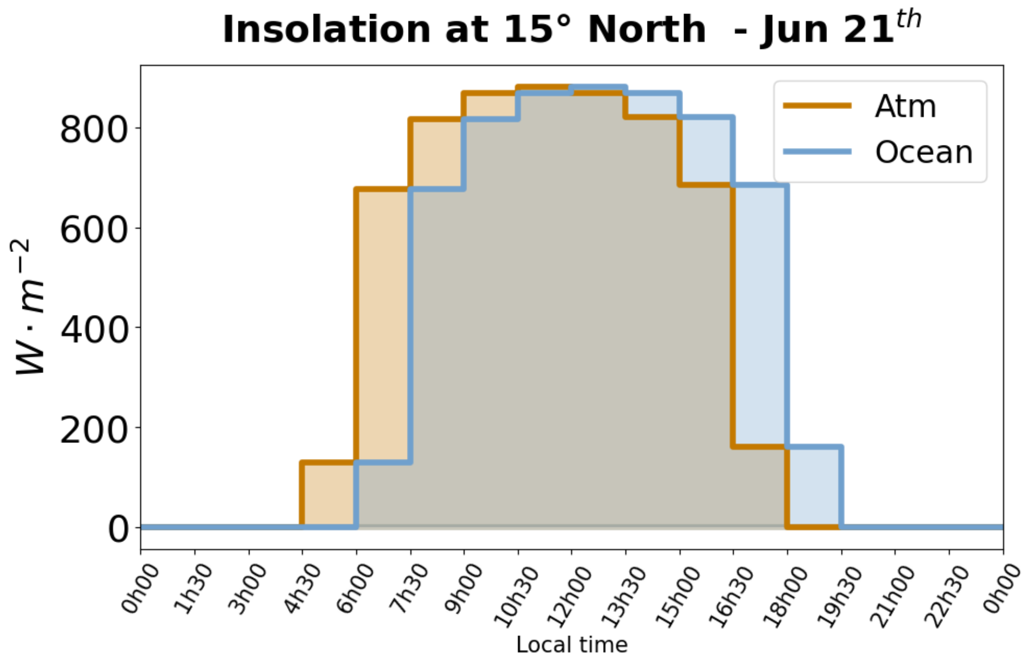
Some models may use an implicit time stepping, but only for vertical diffusion. Not for the full model.

A bit of history

From the origins of IPSL-CM to IPSL-CM5 : coupling once a day. We applied that to a yearly cycle splitted in 360 chunks (days).

IPSLCM6 : coupling time step 1h30. We apply that to a diurnal cycle splitted in 16 chunks ...

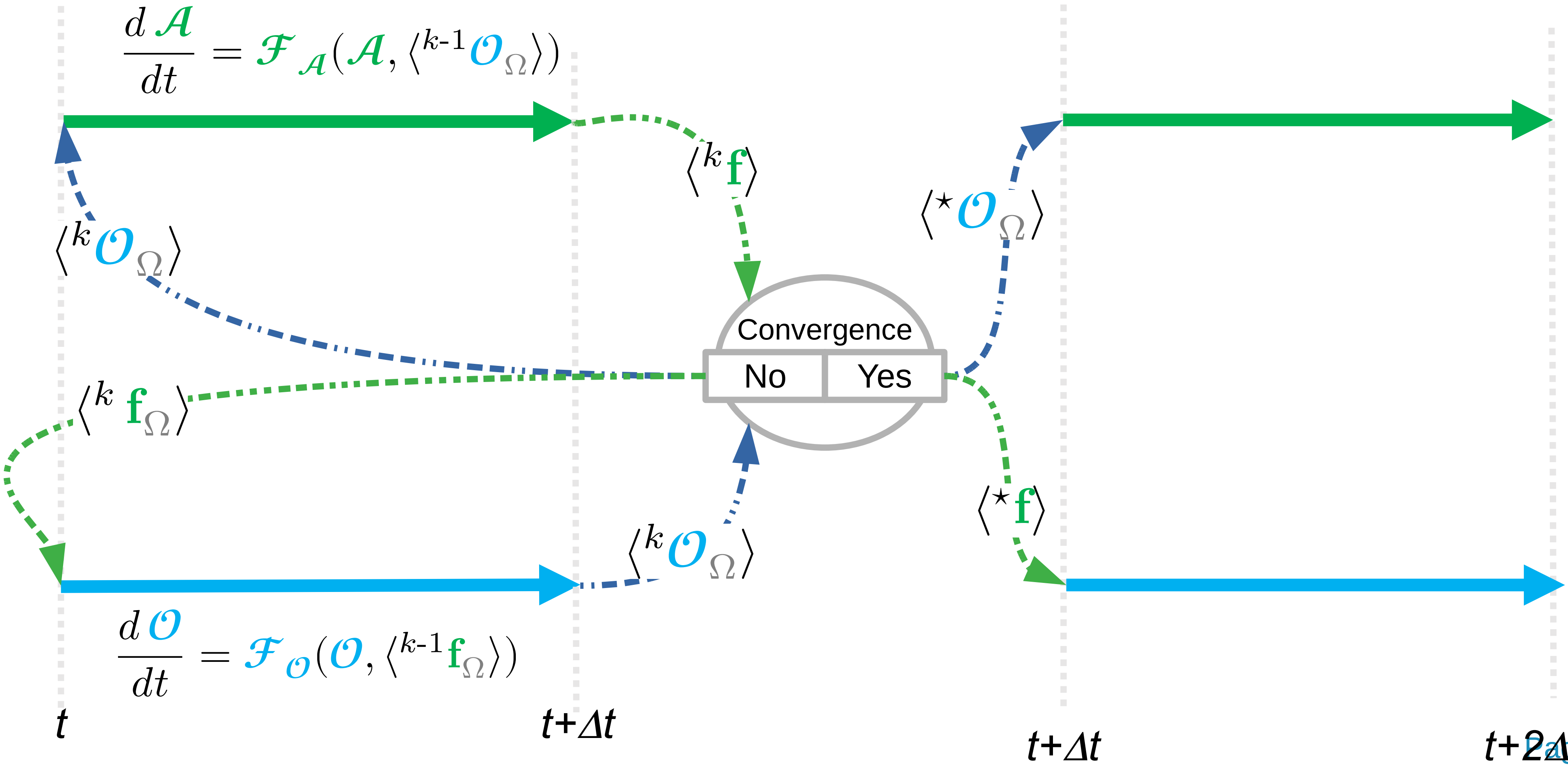




Restoring mathematical consistency : the Schwarz iterative method

Schwarz iterative procedure during one coupling time step [t, $t+\Delta t$]

- Iterations from k=1 to convergence
- For each iteration, initial state of $\mathscr{A}$ and $\mathscr{O}$ are the solutions at the end of the previous time step [$t-\Delta t$, t], when interface values of $\mathscr{O}_\Omega$ and $\mathbf{f}_\Omega$ have converged.
- * denotes the converged solution



The Schwarz iterative method

- Converged solutions $^{*}\mathscr{A}$ and $^{*}\mathscr{O}$

$$\left.\frac{d^{\star}\mathcal{O}}{dt}\right|_{t}^{t+\Delta t} = \mathcal{F}_{\mathcal{O}}\left({}^{\star}\mathcal{O}, \left\langle \mathrm{f}({}^{\star}\mathcal{A}, \langle {}^{\star}\mathcal{O}_{\Omega}\rangle_{t}^{t+\Delta t}) \right\rangle_{t}^{t+\Delta t} \right)$$

- **Re-synchronize components**
- **Mathematically consistent**

- **Tremendous computing cost**

Schwarz in IPSL-CM

Model

Earth System Model IPSL-CM at low resolution (ocean 2°, atmosphere 96x95x39).

Simplified land surface model (bucket)

Sea-ice model : LIM3 monocategory

6 experiments :

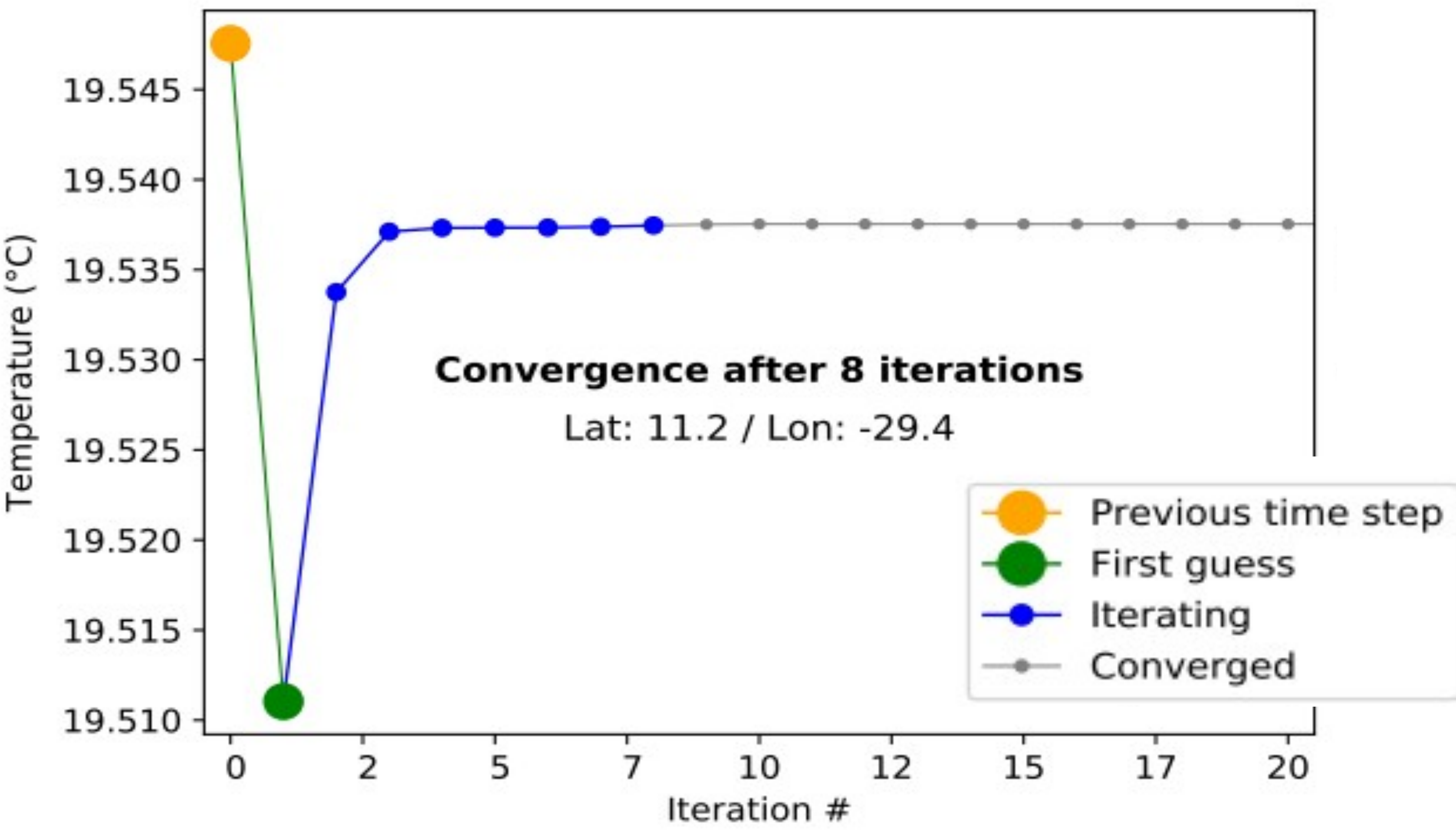
Parallel, Sequential atmosphere first, Sequential ocean first

Coupling time step : Δt = 1h, Δt = 4h

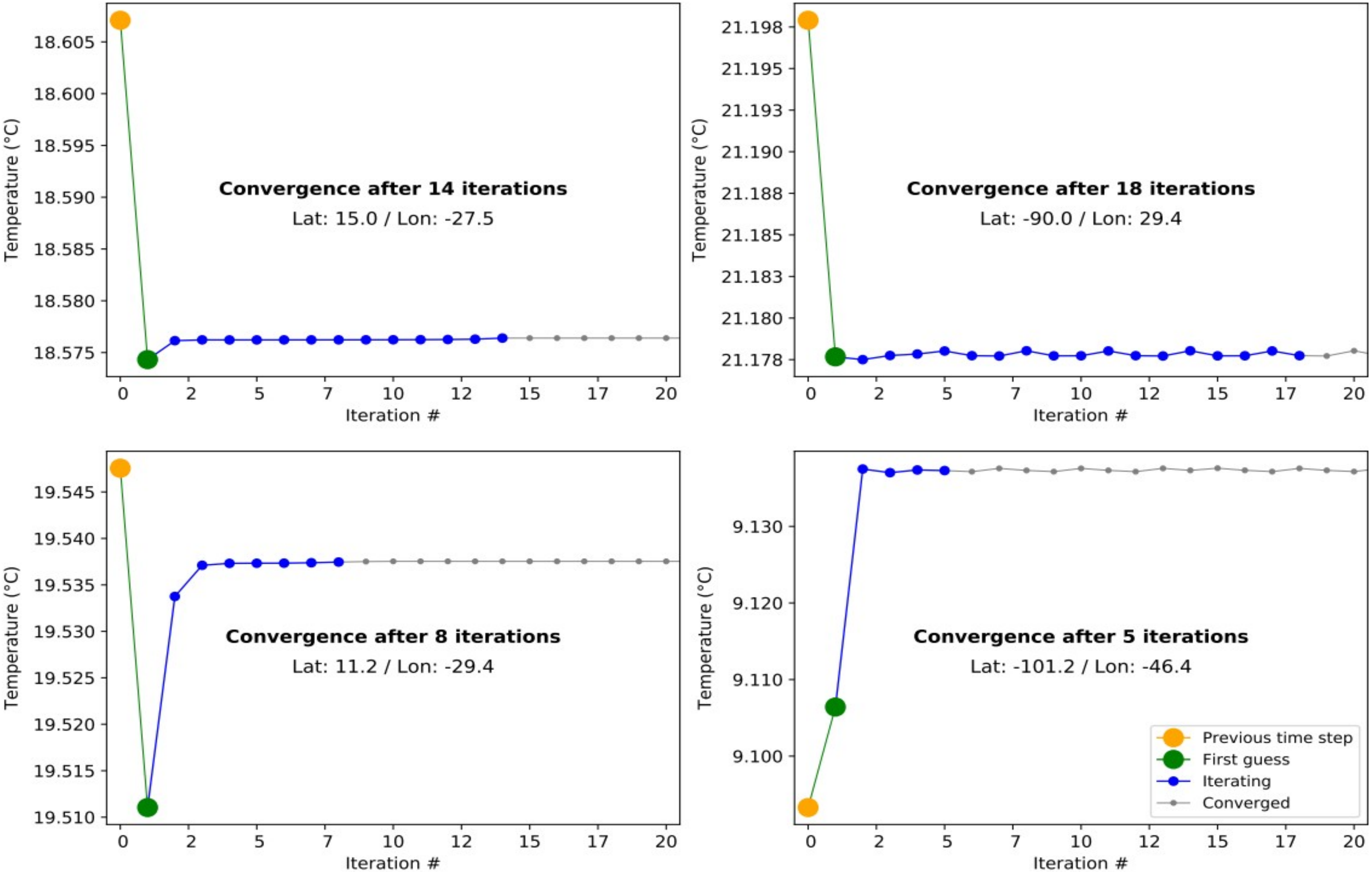
5 days runs

50 iterations

Convergence - SST



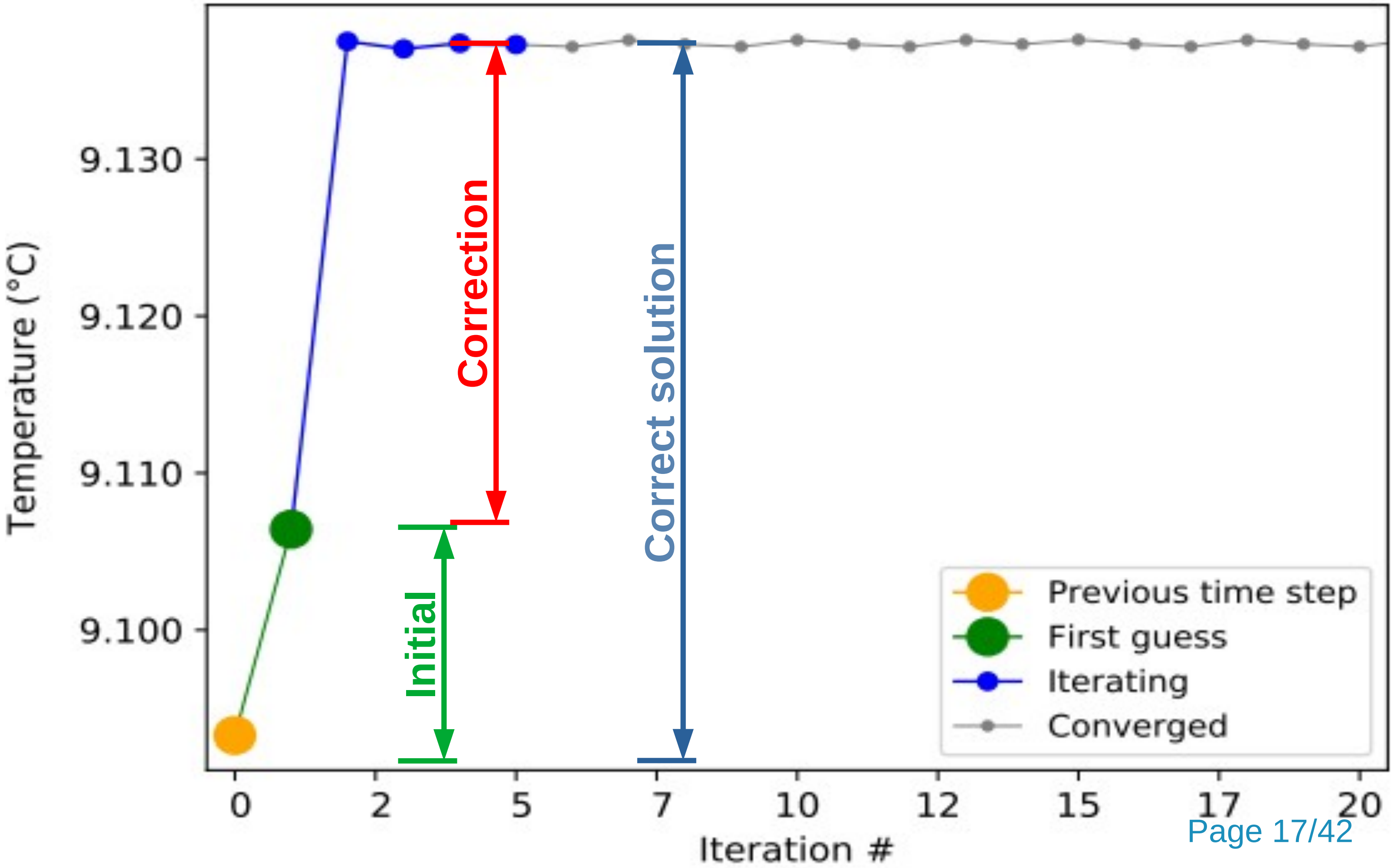
Iterations and convergence on a few selected points



Relative error

Relative error. We consider the ratio between i) the solution change between *t* to *t*+Δ*t* with no Schwarz iteration (**Initial** on the figure), and ii) the correction due to the iterative process (**Correction**).

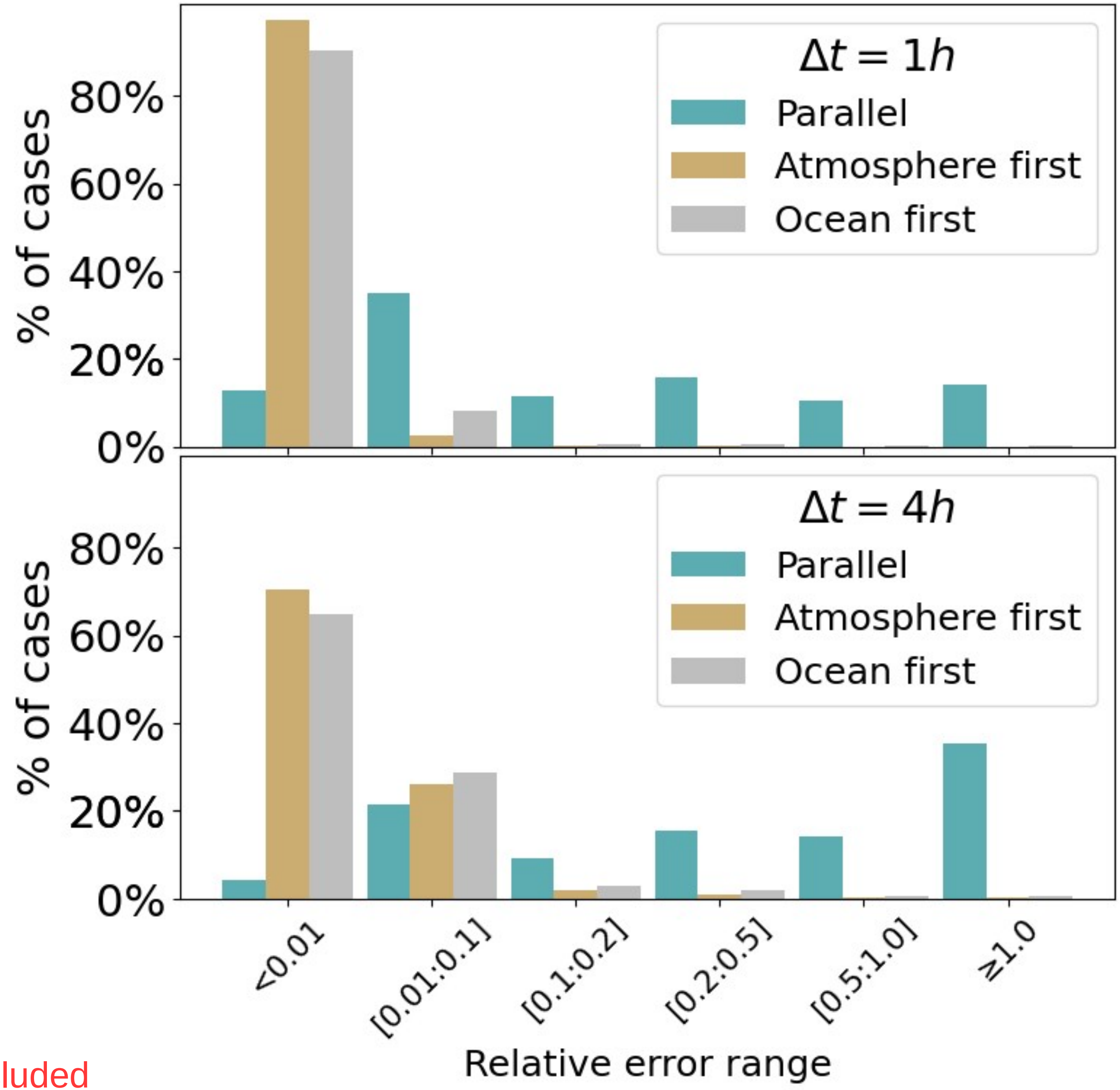
The histogram plots the number of points (in *space* x *time*) for different range of relative error **Correction/Initial**.



Relative error

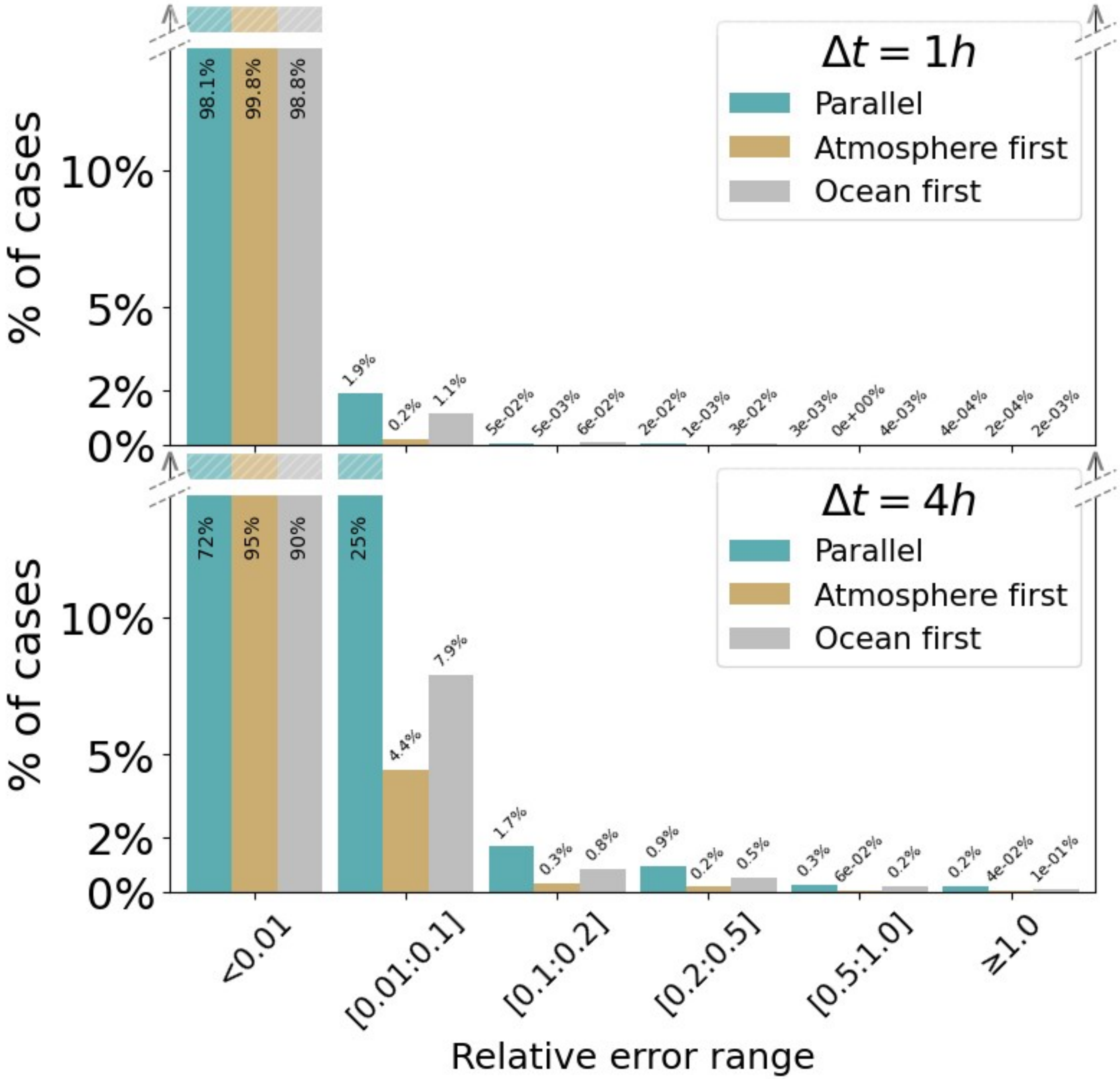
4 688 points x
120 steps =
562 643 cases

4 688 points x
30 steps = 140 k-
cases

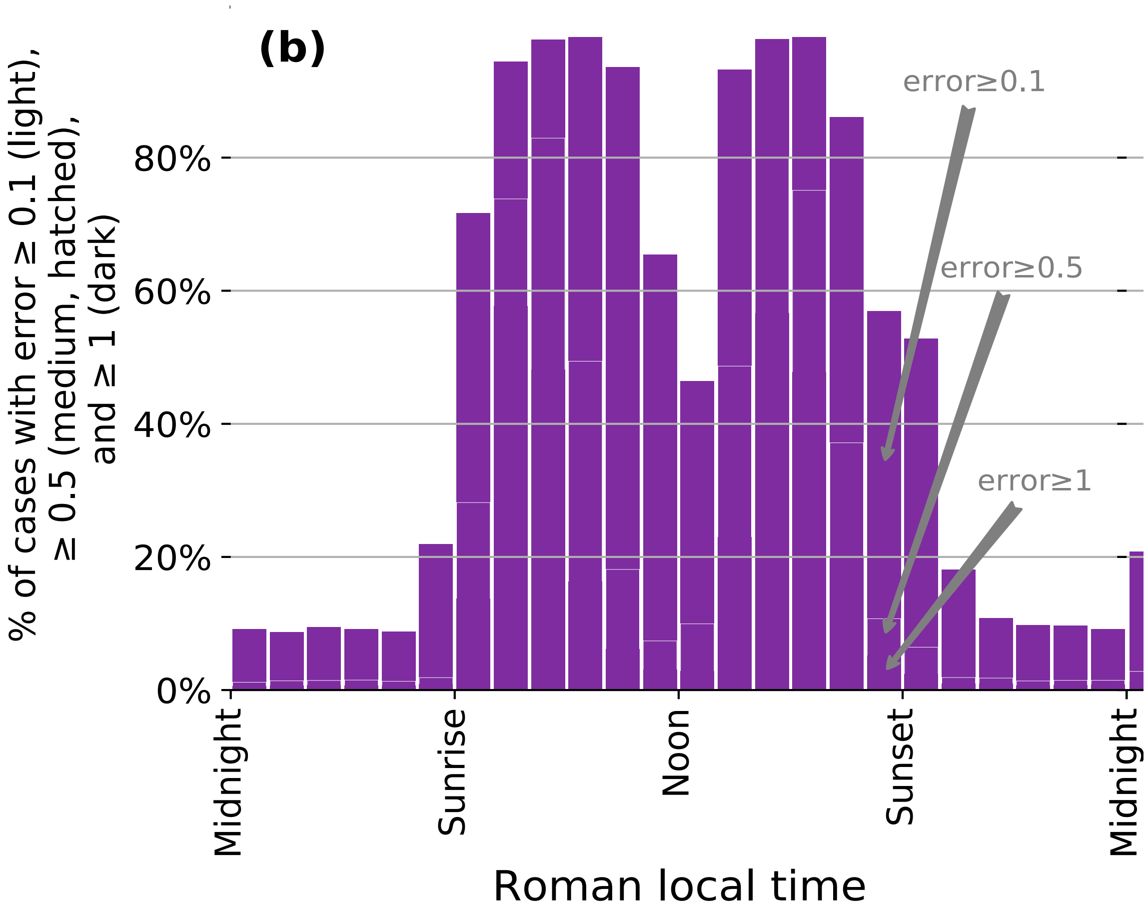


Points with sea-ice are excluded

Relative error after 2 iterations



Diurnal cycle of error



Histograms of errors in function of the local time

Number of cases in each error range (% of total for each local time). Δt = 1h

First conclusions

Current time schemes in state-of-the-art Earth System Model are mathematically inconsistent.

A Schwarz iterative has been implemented in the IPSL coupled model. It is mathematically consistent.

Schwarz iterative method is used as a reference to quantify the error done with the legacy scheme.

This error is quite large.

With a coupling time step Δt = 1, 2 Schwarz iterations can almost cancel the error.

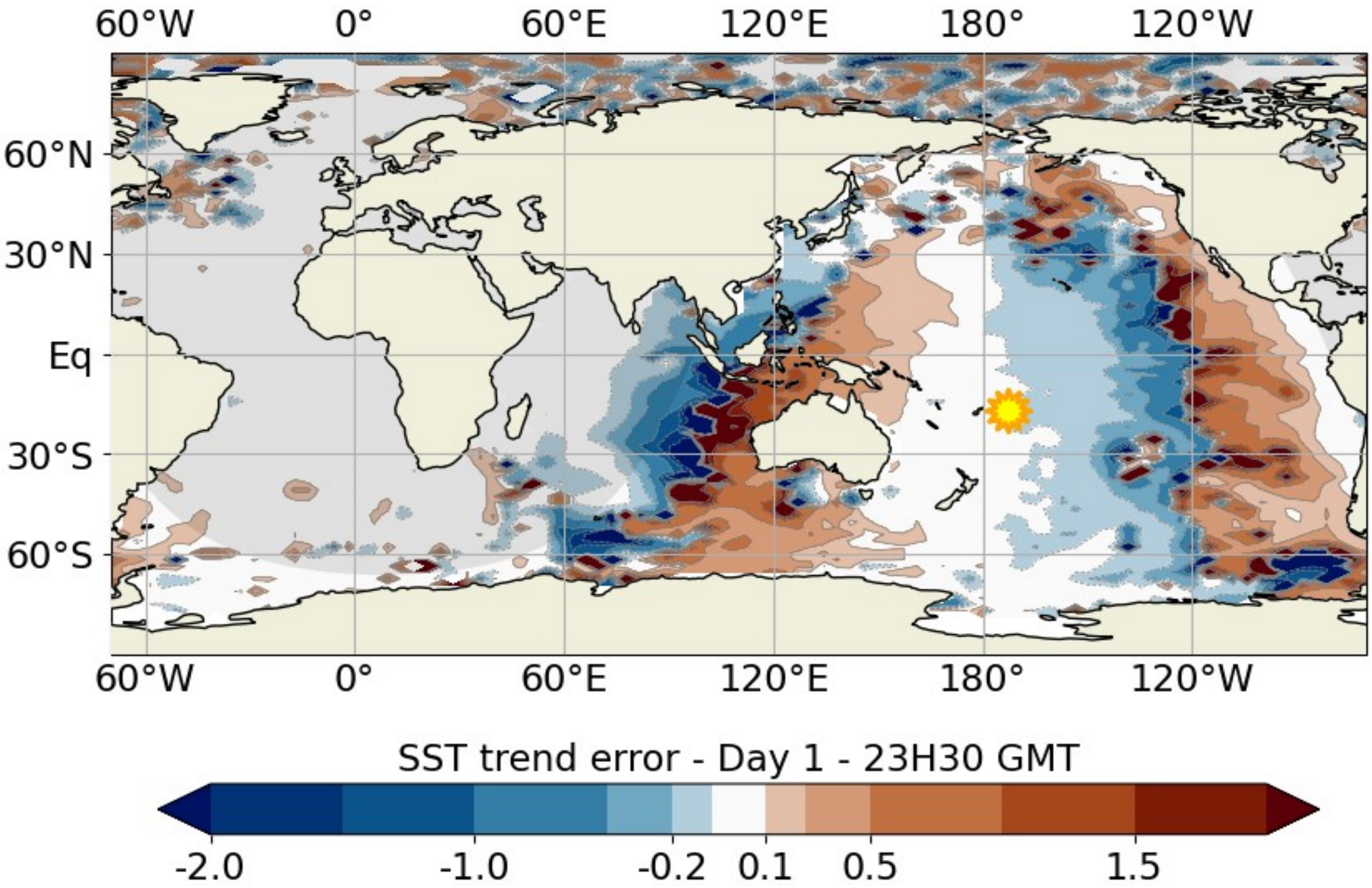
*Sequential atmosphere firs*t has the smaller errors

Marti, O., Nguyen, S., Braconnot, P., Valcke, S., Lemarié, F., and Blayo, E.: A Schwarz iterative method to evaluate ocean–atmosphere coupling schemes: implementation and diagnostics in IPSL-CM6-SW-VLR, Geosci. Model Dev., 14, 2959–2975, https://doi.org/10.5194/gmd-14-2959-2021, 2021.

And for « real » simulations ?

Impact on short experiments

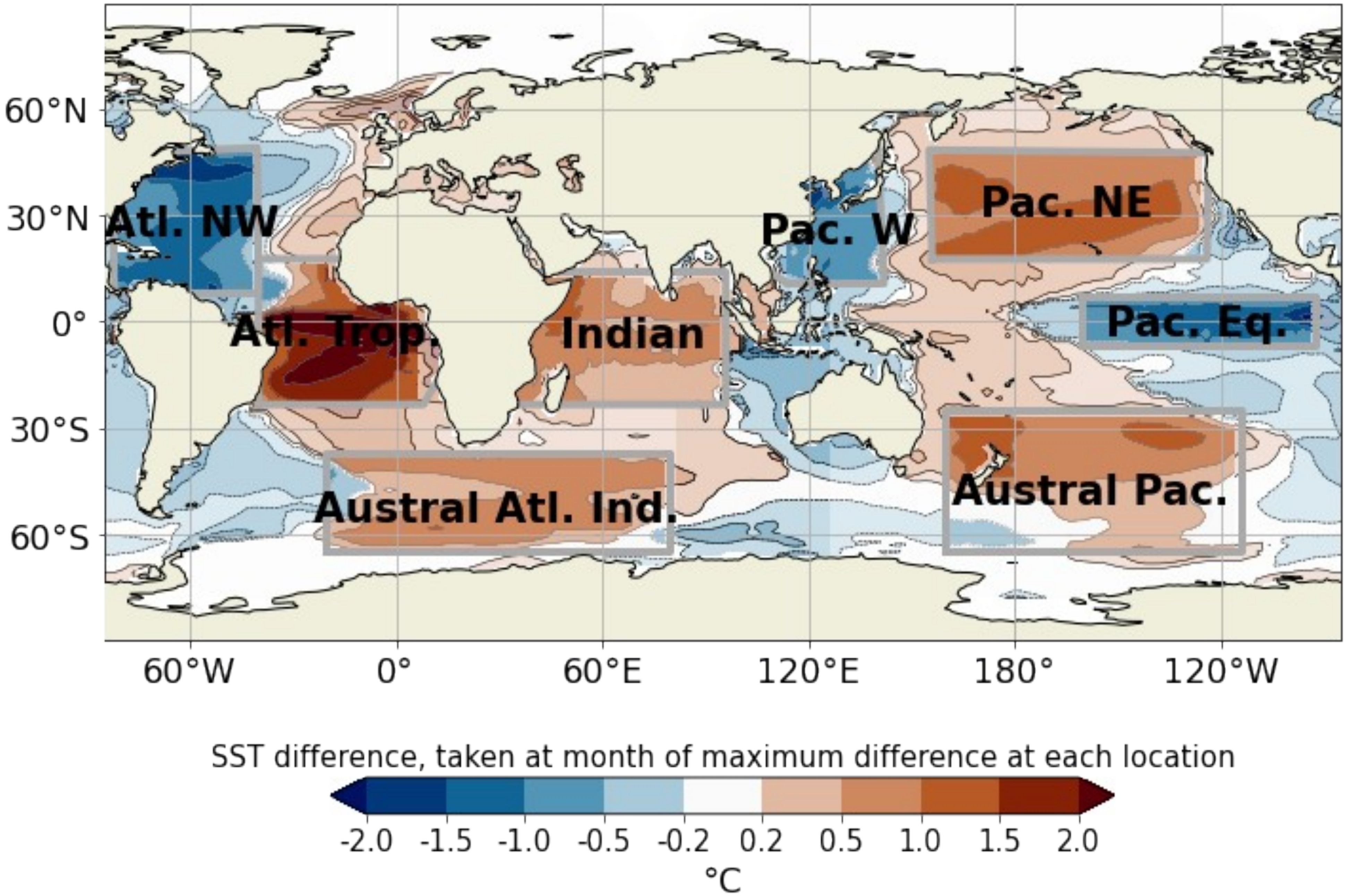
Relative error on SST trend



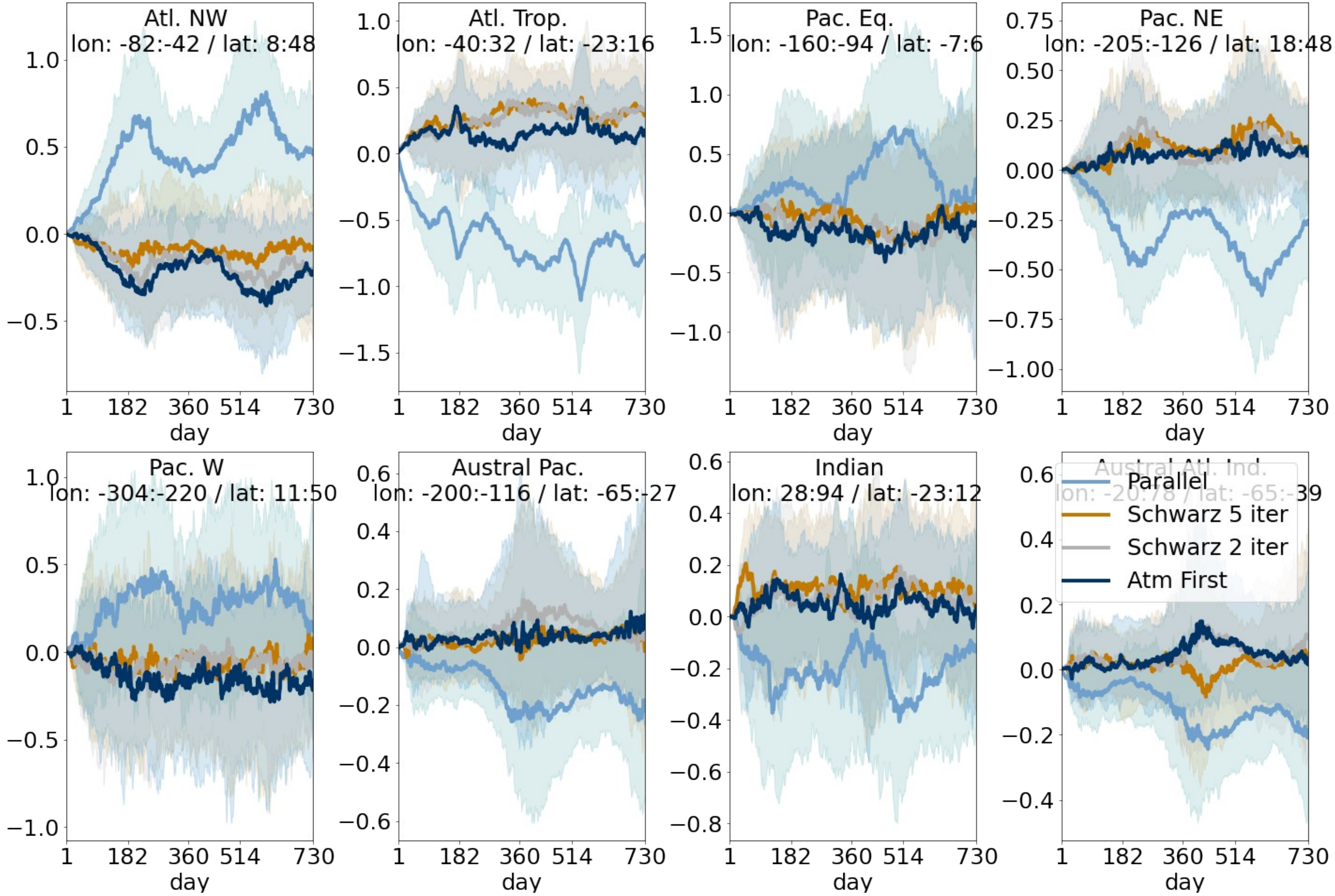
4 experiments

- 2 Years - 50 member
  - Small random pertubation of SST in initial state
- Time schemes
  - Parallel
  - Schwarz 5 iterations
  - Schwarz 2 iterations
  - Sequential Atmosphere First

Difference between Schwarz and Parallel



Daily SST

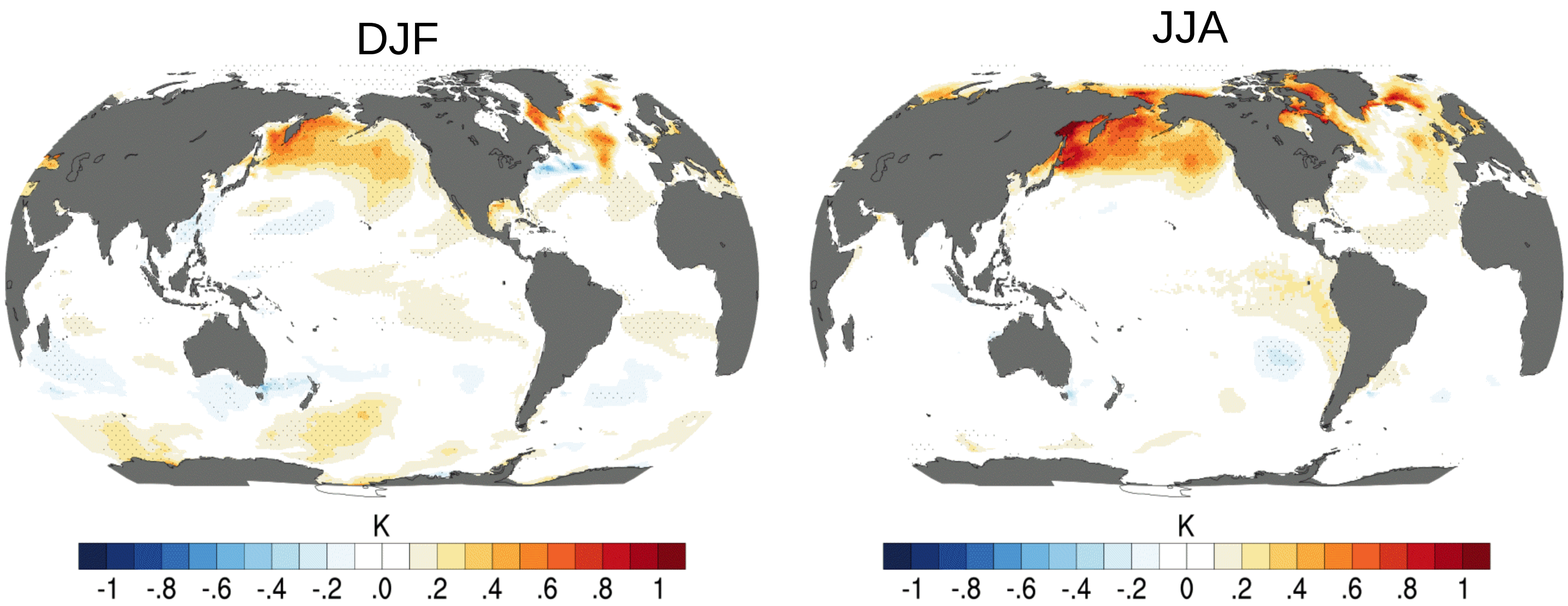


Anomaly with the average of all experiments

Shading - strong: 80% of experiments, light : 90%

Soon : Atmosphere First vs. Parallel in ...

- IPSLMC6-LR (300 years)
- CNRM-CM6-1 (100 years)
- IPSLCM5A2 (300 years)



Difference Seq. Atmosphere first minus Parallel for SST (CNRM-CM6-1)

Conclusions

Current time schemes in state-of-the-art Earth System Model are mathematically inconsistent.

A Schwarz iterative has been implemented in the IPSL coupled model. It is mathematically consistent.

Schwarz iterative method is used as a reference to quantify the error done with the legacy scheme.

This error is quite large.

With a coupling time step $\Delta t =$ of 1 hour, 2 Schwarz iterations can almost cancel the error.

Sequential atmosphere first has the smaller errors

The effect on short experiments is also quite large

Effect on long runs : soon.

The present study focus on the ocean-atmosphere interface, with no sea-ice. The case with three domains (ocean / sea-ice / atmosphere) remains to be investigated. Convergence in presence of sea-ice is very slow.

And now … ?

Study the case with sea-ice

2 Schwarz iterations doubles the cost of the model, which is clearly inacceptable.

Some ideas ?

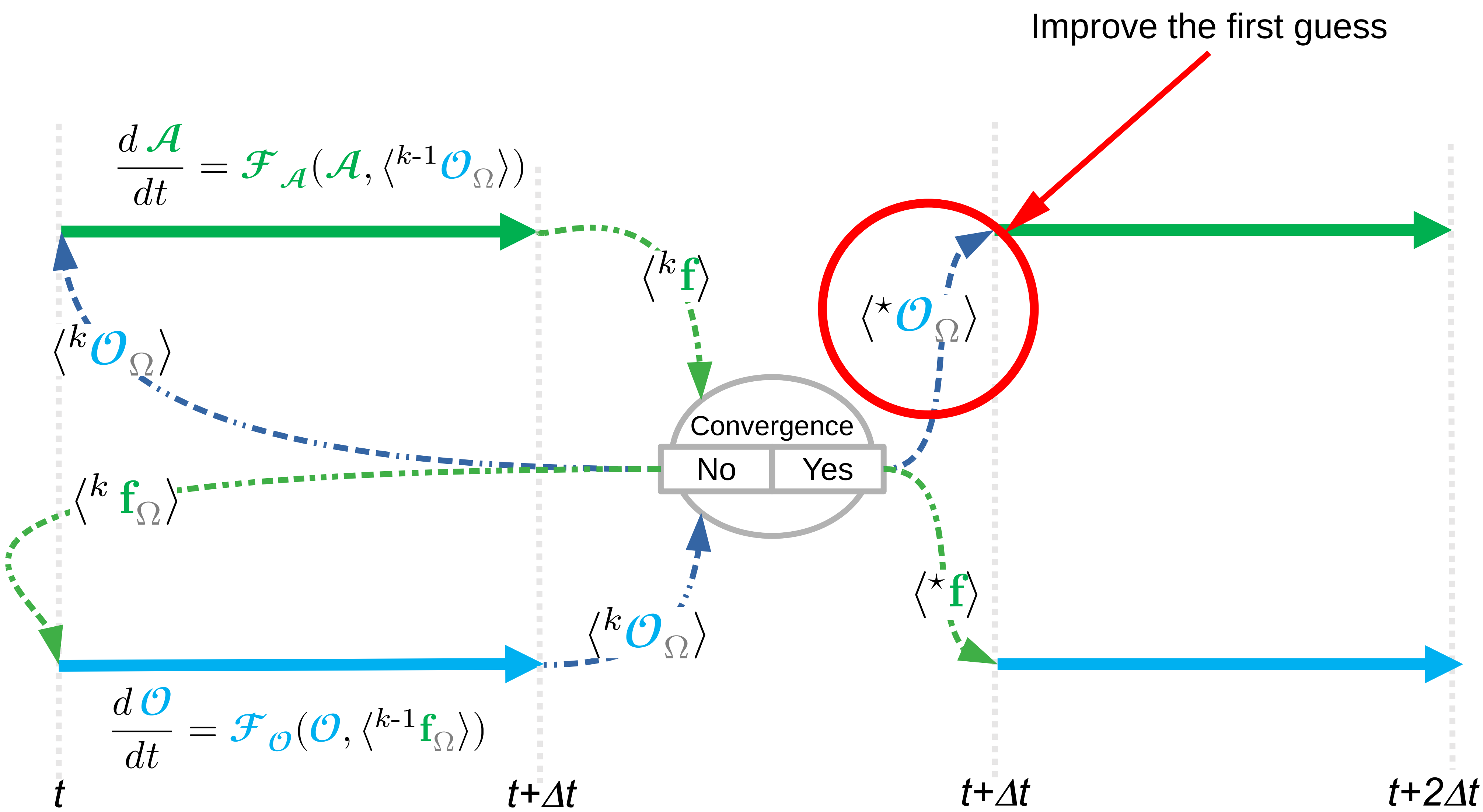
Use *Sequential atmosphere first*.

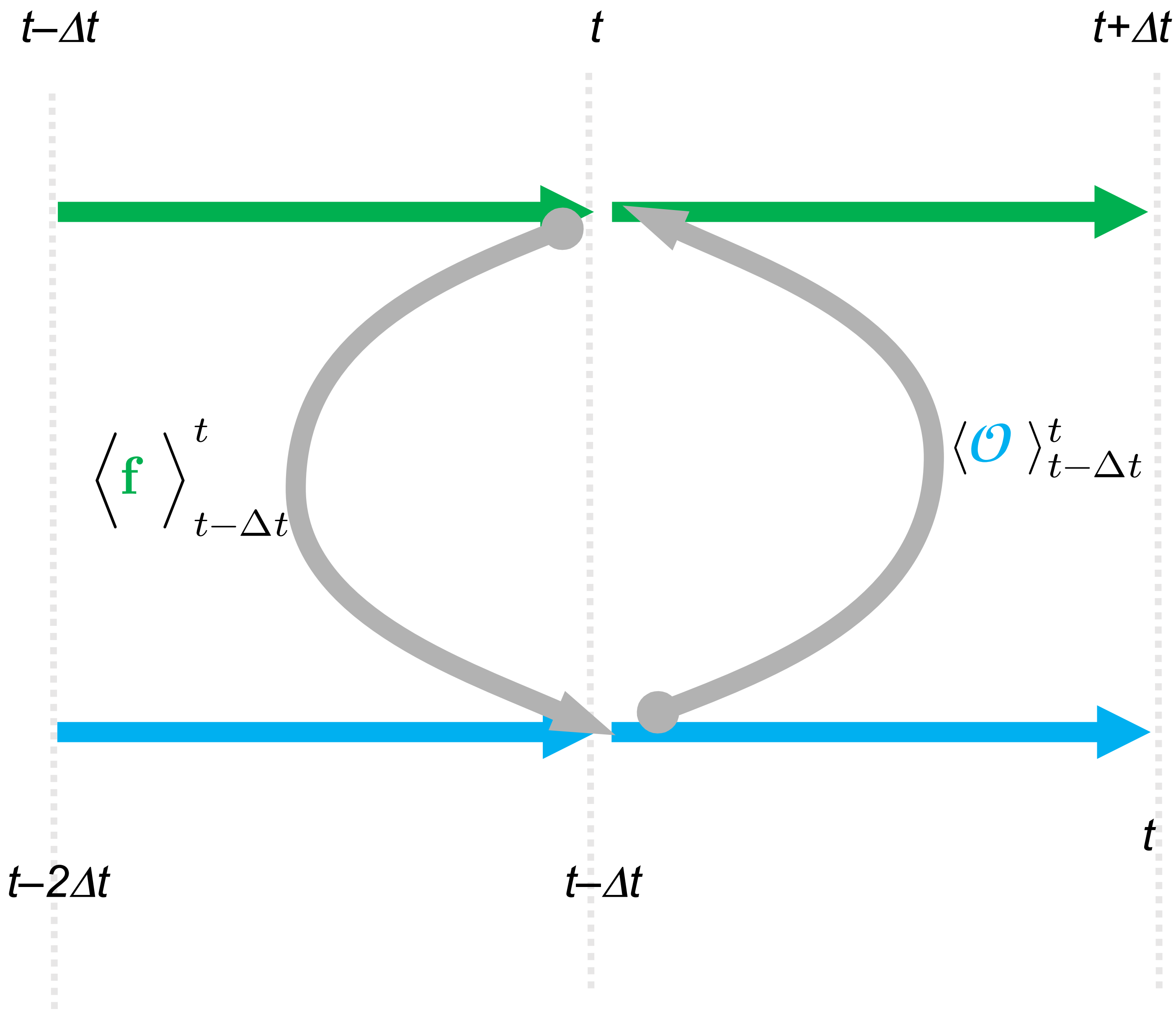
- Increase the time to solution.
- Lot of work to optimize this scheme : memory, load-balancing

Performs Schwarz iterations on a sub part of the model : probably vertical diffusion only.

Improve the first guess to increase the convergence speed

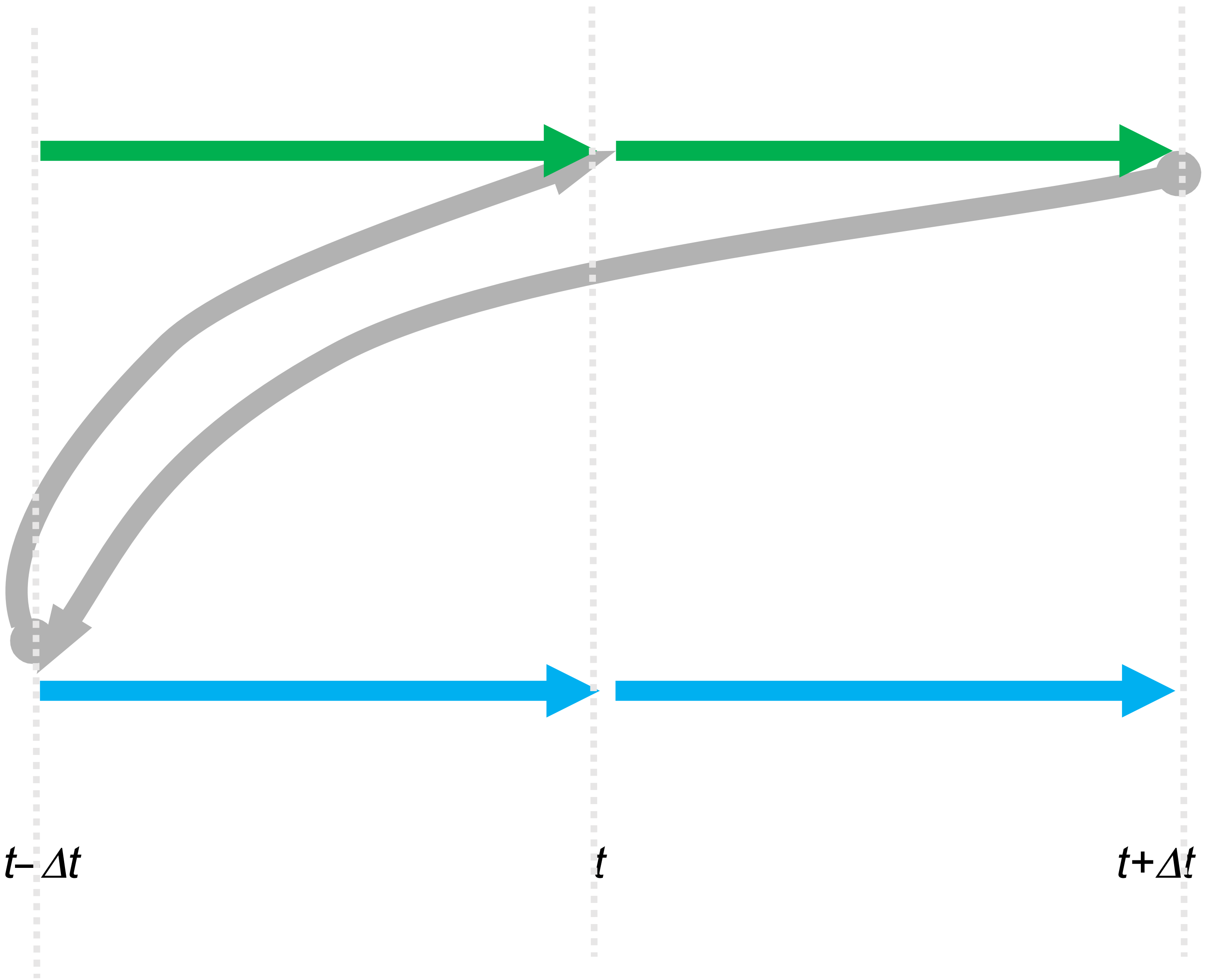
- I.A. ?

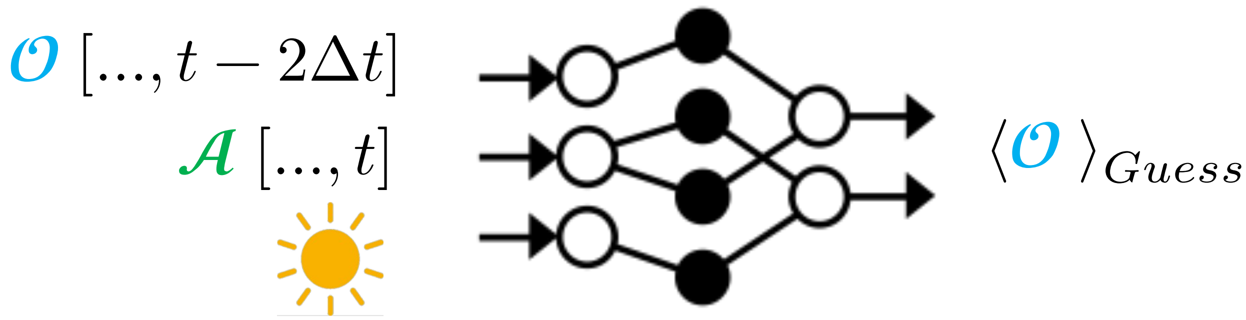

Improve the first guess
$\frac{d\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A}, \langle^{k\text{-}1}\mathcal{O}_{\Omega}\rangle)$
$\langle^{k}\mathbf{f}\rangle$
$\langle^{\star}\mathcal{O}_{\Omega}\rangle$
$\langle^{k}\mathcal{O}_{\Omega}\rangle$
Convergence
No
Yes
$\langle^{k}\mathbf{f}_{\Omega}\rangle$
$\langle^{k}\mathcal{O}_{\Omega}\rangle$
$\langle^{\star}\mathbf{f}\rangle$
$\frac{d\mathcal{O}}{dt} = \mathcal{F}_{\mathcal{O}}(\mathcal{O}, \langle^{k\text{-}1}\mathbf{f}_{\Omega}\rangle)$
t
t+Δt
t+Δt
t+2Δt



$$\left\langle \mathbf{f} \right\rangle_{t}^{t+\Delta t} = \left\langle \mathbf{f}(\mathcal{A}, \langle \mathcal{O} \rangle_{t-2\Delta t}^{t-\Delta t} \right\rangle_{t}^{t+\Delta t}$$

RPN




$\mathcal{O}\,[\ldots, t - 2\Delta t]$
$\mathcal{A}\,[\ldots, t]$
$\langle \mathcal{O} \rangle_{Guess}$

Abstract

State-of-the-art Earth System models, like the ones used in the 6th Coupled Model Intercomparison Project (CMIP6), suffer from mathematical inconsistencies at the ocean-atmosphere interface. Indeed, the coupling algorithms generally implemented in those models do not allow for a correct phasing between the ocean and the atmosphere, and hence between their diurnal cycles. A possibility to remove these temporal inconsistencies is to use an coupling algorithm based on the Schwarz iterative method. This implies a huge increase of the computational cost compared to standard coupling methods, which makes the algorithm implementation impractical as is for production run. However, the Schwarz method is mathematically consistent, and therefore can be use as a reference to evaluate some of the errors made in state-of-the-art ocean-atmosphere coupled models (e.g. in the representation of the processes related to diurnal cycle), as illustrated by the present study.

The Schwarz algorithm is implemented in IPSL-CM6-SW-VLR, a low resolution version of IPSL-CM6 coupled model, with a simplified land surface model. The implementation allows to compare the tendencies of interface variables between the standard scheme and the Schwarz scheme, on a single model trajectories. Comparisons between coupled solutions obtained with this new scheme and the standard IPSL coupling scheme (referred to as *parallel algorithm*) show large differences after sunrise and before sunset, when the external forcing (insolation at top of atmosphere) has the fastest pace of change. At these times of the day, the difference between the two numerical solutions is often larger than 100% of the solution, even with a small coupling period, thus suggesting that significant errors are potentially made with current coupling methods. Two ensembles of 2 year simulations of 50 members each, one with the Schwarz method and one with the legacy *parallel* method, show that the climate computed with the two methods are significantly different, with large differences on some region of the world ocean.

Besides the *parallel* algorithm used in IPSL-CM6, we also test a so-called *sequential atmosphere-first* algorithm which is used in some coupled ocean-atmosphere models. We show that the *sequential* algorithm strongly improves the numerical results compared to the *parallel* one, at the expanse of a loss of parallelism.

Résumé

Les modèles système Terre les plus récents, comme ceux utilisés dans le 6e projet de comparaison des modèles couplés (CMIP6), souffrent d'incohérences mathématiques à l'interface océan-atmosphère. En effet, les algorithmes de couplage généralement utilisés dans ces modèles ne permettent pas un paysage correct entre l'océan et l'atmosphère, et donc entre leurs cycles diurnes. Une possibilité de supprimer ces incohérences temporelles est d'utiliser un algorithme de couplage basé sur la méthode itérative de Schwarz. Cela implique une augmentation considérable du coût de calcul par rapport aux méthodes de couplage standard, ce qui rend la mise en œuvre de l'algorithme peu pratique en l'état pour la production. Cependant, la méthode de Schwarz est mathématiquement cohérente, et peut donc être utilisée comme référence pour évaluer certaines des erreurs commises dans les modèles de couplage océan-atmosphère de pointe (par exemple, dans la représentation des processus liés au cycle diurne), comme l'illustre la présente étude.

L'algorithme de Schwarz est implémenté dans IPSL-CM6-SW-VLR, une version basse résolution du modèle couplé IPSL-CM6, avec un modèle de surface terrestre simplifié. L'implémentation permet de comparer les tendances des variables d'interface entre le schéma standard et le schéma de Schwarz, sur les trajectoires d'un seul modèle. Les comparaisons entre les solutions couplées obtenues avec ce nouveau schéma et le schéma de couplage standard IPSL (appelé algorithme parallèle) montrent de grandes différences après le lever et avant le coucher du soleil, lorsque le forçage externe (insolation au sommet de l'atmosphère) a le rythme de changement le plus rapide. À ces moments de la journée, la différence entre les deux solutions numériques est souvent supérieure à 100 % de la solution, même avec une petite période de couplage, ce qui suggère que des erreurs importantes sont potentiellement commises avec les méthodes de couplage actuelles. Deux ensembles de simulations sur 2 ans de 50 membres chacun, l'un avec la méthode Schwarz et l'autre avec la méthode parallèle héritée, montrent que le climat calculé avec les deux méthodes est significativement différent, avec de grandes différences sur certaines régions de l'océan mondial.

Outre l'algorithme parallèle utilisé dans IPSL-CM6, nous testons également un algorithme dit séquentiel atmosphère-première qui est utilisé dans certains modèles couplés océan-atmosphère. Nous montrons que l'algorithme séquentiel améliore fortement les résultats numériques par rapport à l'algorithme parallèle, au prix d'une perte de parallélisme.